Problem Set 2
Optical Waveguides and Fibers (OWF)

will be discussed in the tutorial on November 11, 2015

Exercise 1: Dispersion of fused silica (amorphous SiO_2)

The refractive indices of dielectric materials can be written in functional form by means of the Sellmeier equations. For fused silica, the most common material used for fabricating optical fibers, the Sellmeier equation takes the following form:

$$n^2(\lambda) = 1 + \frac{0.6962\lambda^2}{\lambda^2 - (0.06840)^2} + \frac{0.4079\lambda^2}{\lambda^2 - (0.1162)^2} + \frac{0.8975\lambda^2}{\lambda^2 - (9.8962)^2}. \tag{1}$$

The quantity λ denotes the vacuum wavelength in micrometers.

a) Generate a computer plot, e.g., using MATLAB, that shows the refractive index of fused silica as a function of wavelength. Eq. (1) is valid between $0.2\,\mu$m and $3.7\,\mu$m, i.e., from the ultraviolet region to the near infrared.

Hint: MATLAB can be accessed from any computer at the SCC. For home use, a licence can be downloaded by any student via the SCC: http://www.scc.kit.edu/produkte/3841.php.

b) Consider a pulse of light with center wavelength λ propagating over 1 km through bulk fused silica. Plot the arrival time as a function of λ.

c) Short pulses have broad spectra, i.e., they consist of various different wavelength components. Which center wavelength would you choose to transmit a short pulse through bulk fused silica with minimum impairment?

d) Plot the material dispersion coefficient M_λ as a function of wavelength on a scale having the units $\frac{\text{ps}}{\text{km nm}}$, which are the most common units for this quantity.

Exercise 2: Spreading of a Gaussian pulse as it propagates in a dispersive medium.

Consider a pulse which is propagating along the z direction within a material having material dispersion M_λ at the carrier angular frequency ω_c. Assume that at $z = 0$ the pulse is described by:

$$\underline{a}(z=0,t) = \underline{A}_0 e^{-\frac{t^2}{2\sigma_t^2(0)}} e^{\mathrm{j}\omega_c t} \tag{2}$$

a) Calculate $\underline{a}(z,t)$ for $z > 0$. To do so, you can proceed in the following way:

- Calculate the Fourier transform of the pulse.
- Assume a complex propagator of the form $e^{-\mathrm{j}\beta(\omega)z}$. Use the Taylor expansion up to second order to approximate the propagation constant, i.e., $\beta(\omega) = \beta_c + \beta_c^{(1)}(\omega-\omega_c) + \frac{1}{2}\beta_c^{(2)}(\omega-\omega_c)^2$.
- Perform the inverse Fourier transform. Hint: Introduce the quantity $\underline{\sigma}_t^2(z) = \sigma_t^2(0) + \mathrm{j}\beta_c^{(2)}z$

b) Show that the pulse remains Gaussian and that

$$|\underline{a}(z,t)| \propto e^{-\frac{\left(t-\beta_c^{(1)}z\right)^2}{2\sigma_t^2(z)}}, \tag{3}$$

where

$$\sigma_t^2(z) = \sigma_t^2(0) + \frac{\left(\beta_c^{(2)}z\right)^2}{\sigma_t^2(0)}. \tag{4}$$

c) How do $\beta_c^{(0)}$, $\beta_c^{(1)}$ and $\beta_c^{(2)}$ influence the optical signal?

Note 1: Fourier transform convention
Remember that in this course we use the following definition of the Fourier transform.

$$\mathcal{F}\left[\underline{f}(t)\right] = \underline{\tilde{f}}(\omega) = \int_{-\infty}^{+\infty} \underline{f}(t) e^{-\mathrm{j}\omega t} \mathrm{d}t \tag{5}$$

The latter equation implies that the inverse transform is:

$$\mathcal{F}^{-1}\left[\underline{\tilde{f}}(\omega)\right] = \underline{f}(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \underline{\tilde{f}}(\omega) e^{\mathrm{j}\omega t} \mathrm{d}\omega \tag{6}$$

Note 2: Fourier transform of the Gaussian function

According to the previous definition:

$$\mathcal{F}\left[e^{-\frac{t^2}{2\sigma^2}}\right] = \sqrt{2\pi}\sigma e^{-\frac{\sigma^2\omega^2}{2}}, \text{ for } \mathrm{Re}\left[\frac{1}{\sigma^2} > 0\right]. \tag{7}$$

Note 3: Relation between the Taylor expansion of $\beta(\omega)$ and the material dispersion M_λ

The definition of the material dispersion coefficient M_λ is:

$$M_\lambda = \frac{\Delta t_g}{z \Delta \lambda}. \tag{8}$$

This relation can be easily remembered when recalling the previously introduced dimension, $\frac{\text{ps}}{\text{km nm}}$: M_λ gives the group delay spread Δt_g in ps between two wavepackets for which the center wavelengths are separated by $\Delta\lambda = 1\text{nm}$ after a propagation distance of $z = 1\text{km}$.
A useful relation between M_λ and $\beta_c^{(2)}$ can be obtained when using

$$t_g \equiv \frac{z}{v_g} = z\beta_c^{(1)}, \tag{9}$$

in Eq. (8)

$$M_\lambda = \frac{\mathrm{d}\beta_c^{(1)}}{\mathrm{d}\lambda} = -\frac{\omega}{\lambda} \frac{\mathrm{d}\beta_c^{(1)}}{\mathrm{d}\omega} = -\frac{2\pi c}{\lambda^2} \beta_c^{(2)}. \tag{10}$$

Questions and Comments:

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